1 Introduction to Basic Geometry

1.1 Euclidean Geometry and Axiomatic Systems

1.1.1 Points, Lines, and Line Segments

Geometry is one of the oldest branches of mathematics. The word geometry in the Greek language translates the words for "Earth" and "Measure". The Egyptians were one of the first civilizations to use geometry. The Egyptians used right triangles to measure and survey land. In our modern times, geometry is used to in fields such as engineering, architecture, medicine, drafting, astronomy, and geology. To begin this chapter on Geometry, we will describe two basic concepts which are a point and a line. A point is used to denote a specific location in space. In this section, everything that we do will be viewed in two dimensions. For example, we could draw a point in two dimensional space and label it as point A.

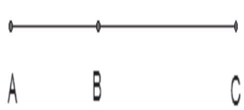
A line is determined by two distinct points and extends to infinity in both directions. Now suppose that we define two points in space and label them as A and B. We could pass a line through these points in space and the resulting line would look the next illustration. We will label this line as $\overrightarrow{AB}$

A line segment is part of a line that lies between two points. These two points are referred to as endpoints. In the next figure below there is an illustration of a line segment. We will label this line segment as $\overline{AB}$

1.1.2 Distance

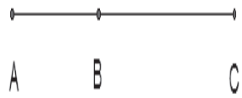
Now that we have given a basic description a line and line segment, let's use some properties of distance to find the missing length of a segment. In the next example we will find the distance between two points. To find missing distances of a line segment, we use a postulate called the segment addition postulate.

Segment Addition Postulate
If point B lies between points A and C on $\overline{AB}$, then AB + BC = AC



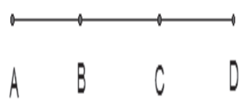
In the next example will find the distance between two points.

Example 1
Given AB = 2x + 3, BC = 3x +7, and AC = 25, find the value of x, AB, and BC



Solution
Since the point B lies between point A and C on $\overline{AC}$, it must be true that AB + BC = AC. Substituting the values for AB, BC and AC into the above equation, we get the following equation that can be solved for x. $2x + 3 + 3x + 7 = 25$
$5x + 10 = 25$
$5x + 10 - 10 = 25 - 10$
$5x = 15$
$\frac{5x}{5} = \frac{15}{5}$
$x = 3$
Now, use the value of x to find the values of and AB and BC.
Therefore, $AB = 2(3) + 3 = 6 + 3 = 9$ and $BC = 3(3) + 7 = 9 + 7 = 16$

Example 2
Given AB = 10, BC = 2x + 4, CD = 12, and AD = 36, find the length of BC.

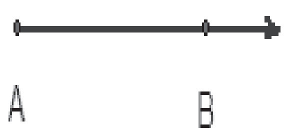


Since points B and C lie between points A and D on $\overline{AD}$, AB + BC + CD = AD
$AB + BC + CD = AD$
$10 + 2x + 4 + 12 = 36$
$2x + 26 = 36$
$2x + 26 - 26 = 36 - 26$
$2x = 10$
$x = 5$

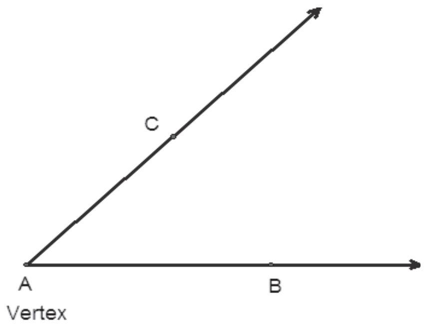
Now, use the value of x to find the length of BC: $BC = 2(5) + 4 = 10 + 4 = 14$

1.1.3 Rays and Angles

A ray starts at a point called an endpoint and extends to infinity in the other direction. A ray that has an endpoint at A and extends indefinitely through another point B is denoted by $\overrightarrow{AB}$ Here is an example of a ray with an endpoint A that lies in a plane.



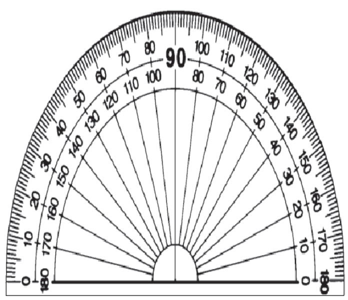
An angle is the union of two rays with a common endpoint called a vertex.



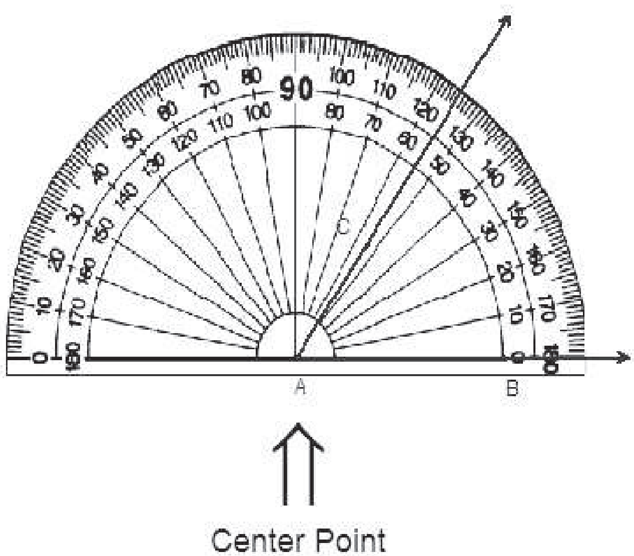
In the diagram above the vertex of the angle is A.

1.1.4 Angle Measure

Angles can be measured in degrees and radians. In Euclidean Geometry angles are measured in degrees and usually the smallest possible angle is 0 degrees and the largest possible angle is 180 degrees. Let's briefly discuss how to measure angles using degrees. The most common way to measure angles can is by a protractor. A protractor, shown below, is a devise use to measure angles.



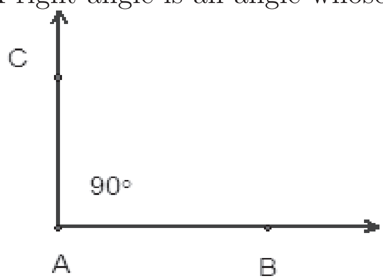
To measure an angle using a protractor, you place the protractor over the angle and line up the center point of the protractor up with the vertex of the angle as shown in next diagram. Next, you find the side of the angle that isn't lined up with the base of the protractor and read the angle measure from the protractor.



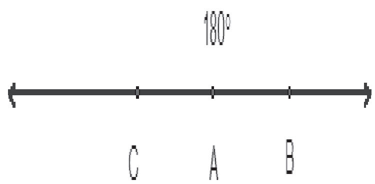
The measure of the angle in the above diagram would be 55 degrees.

1.1.5 Special Types of Angles

A right angle is an angle whose measure is 90 degrees.

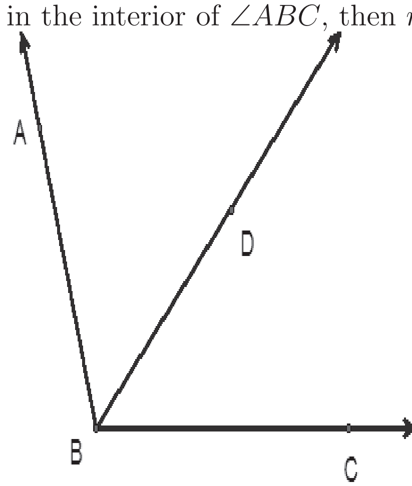


A straight angle is an angle whose measure is 180 degrees.



Special Angle Pairs
There are two types of angle pairs which are complementary angles and supplementary angles. A pair of complementary angles are two angles whose sum is 90 degrees. Meanwhile, A pair of supplementary angles are two angles whose sum is 180 degrees. Adjacent Angles are two angles who share a common endpoint and common side, but share no interior points.

The Angle Addition Postulate
If point D lies in the interior of $\angle ABC$, then $m\angle ABC = m\angle ABD + m\angle DBC$



1.1.6 Finding Missing Angle Values

To find the value of the missing angle, we will use the angle addition postulate along with the definition of complementary angles and supplementary angles.

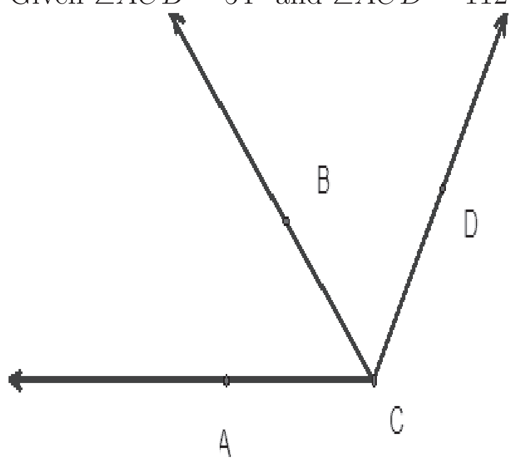
Example 3
Find the complement of angle measuring 36^0

Solution
If the angles are complementary, their sum is 90 degrees. Now, let are missing angle be $\angle A$
Therefore, $\angle A + 36^0 = 90^0 \rightarrow \angle A = 90^0 - 36^0 = 54^0$

Example 4
Find the supplement of angle measuring 86^0

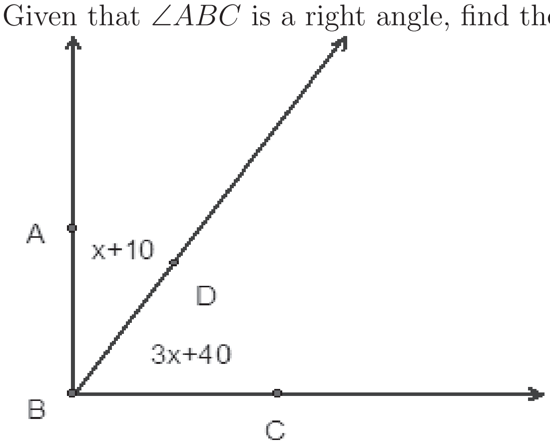
Solution
If the angles are supplementary, their sum is 180 degrees. Now, let are missing angle be $\angle A$
Therefore, $\angle A + 86^0 = 180^0 \rightarrow \angle A = 180^0 - 86^0 = 94^0$

Example 5
Given $\angle ACB = 54^0$ and $\angle ACD = 112^0$, find the measure of $\angle ACD$



Solution $\angle BCD$ can be found by subtracting $\angle ACB$ from $\angle ACD$
$\angle BCD = 112^0 - 54^0 = 58^0$

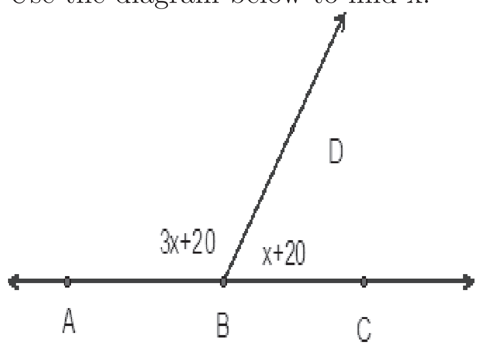
Example 6
Given that $\angle ABC$ is a right angle, find the value of x, $\angle ABD$, $\angle DBC$



Solution:

Set the sum of the angles equal to 90 degrees and solve the resulting equation for x.
$m\angle ABD + m\angle DBC = 90^0$
$x + 10^0 + 3x + 40^0 = 90^0$
$4x + 50^0 = 90^0$
$4x + 50^0 - 50^0 = 90^0 - 50^0$
$4x = 40^0$
$\frac{4x}{4} = \frac{40}{4}$
$x = 10$
Now, use the value of x to find the measures of $\angle ABD$ and $\angle DBC$
$\angle ABD = 10^0 + 10^0 = 20^0$
$\angle DBC = 3(10^0) + 40^0 = 30^0 + 40^0 = 70^0$

Example 7
Use the diagram below to find x.



Solution
The angle pair above is supplementary so their sum is 180 degrees. Use the angle addition postulate to find x.
$3x + 20 + x + 20 = 180$
$4x + 40 = 180$
$4x + 40 - 40 = 180 - 40$
$4x = 140$
$\frac{4x}{4} = \frac{140}{40}$
$x = 35$

1.1.7 Axiomatic Systems

An Axiomatic system is a set of axioms from which some or all axioms can be used in conjunction to logically derive a system of Geometry. In an axiomatic system, all the axioms that are defined must be consistent where there are no contractions within the set of axioms. The first mathematician to design an axiomatic system was Euclid of Alexandria. Euclid of Alexandria was born around 325 BC. Most believe that he was a student of Plato. Euclid introduced the idea of an axiomatic geometry when he presented his 13 chapter book titled The Elements of Geometry. The Elements he introduced were simply fundamental geometric principles called axioms and postulates. The most notable are Euclid five postulates which are stated in the next passage.

1. Any two points can determine a straight line.

2. Any finite straight line can be extended in a straight line.

3. A circle can be determined from any center and any radius.

4. All right angles are equal.

5. If two straight lines in a plane are crossed by a transversal, and sum the interior angle of the same side of the transversal is less than two right angles, then the two lines extended will intersect.

According to Euclid, the rest of geometry could be deduced from these five postulates. Euclid's fifth postulate, often referred to as the Parallel Postulate, is the basis for what are called Euclidean Geometries or geometries where parallel lines exist. There is an alternate version to Euclid fifth postulate which is usually stated as "Given a line and a point not on the line, there is one and only one line that passed through the given point that is parallel to the given line." This is a short version of the Parallel Postulate called Fairplay's Axiom which is named after the British math teacher who proposed to replace the axiom in all of the schools textbooks. Some individuals have tried to prove the parallel postulate, but after more than two thousand years it still remains unproven. For many centuries, these postulates have assumed to be true. However, some mathematics believed that the Euclid Fifth Postulate was suspect or incomplete. As a result, mathematicians have written alternate postulates to the Parallel Postulate. These postulates have led the way to new geometries called Non-Euclidean Geometries.

1.1.8 Exercises

1. Which angle represents the compliment of 54^0?

   (A) 126^0
   (B) 46^0
   (C) 36^0
   (D) 26^0

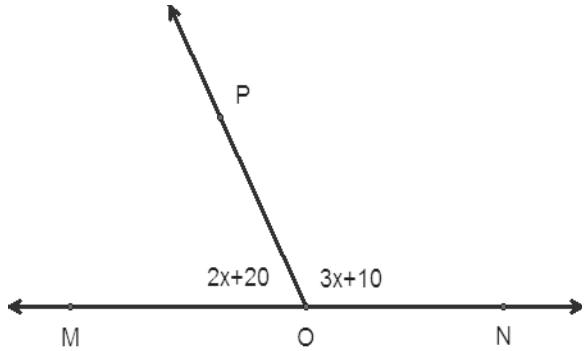
2. What angle represents the supplement of 65^0?

   (A) 125^0
   (B) 135^0
   (C) 25^0
   (D) 35^0

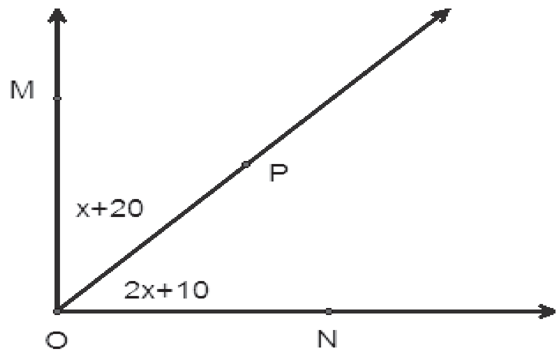
3. Find the supplement of 45^0?

4. Find the supplement of 45^0?

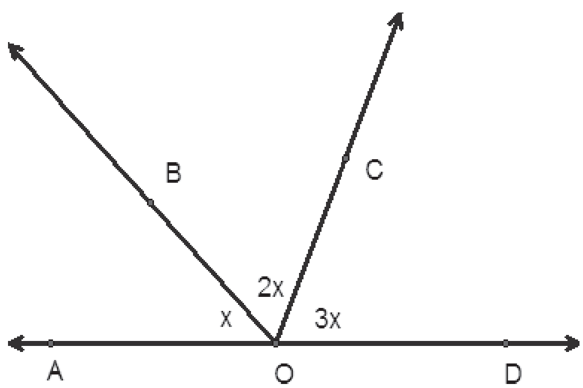
5. Use the diagram below to find the value of x.



6. Given that $\angle MON$ is a right angle, find the measure of $\angle MOP$ and $\angle PON$



7. Use the diagram below to find the value of x.

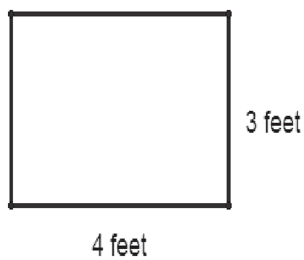


1.2 Perimeter and Area

1.2.1 Understanding Perimeter

We encounter two dimensional objects all the time. We see objects that take on the shapes similar to squares, rectangle, trapezoids, triangles, and many more. Did you every think about the properties of these geometric shapes? The properties of these geometric shapes include perimeter, area, similarity, as well as other properties. The first of these properties that we will investigate is perimeter. The perimeter of an object can be thought of as the distance around the object. In case of an object such as a square, rectangle, or triangle, the perimeter of an object can be found by taking the sum of the sides of the object. Here is an simple example of finding the perimeter of an object.

Example 1
Find the perimeter of the rectangle.



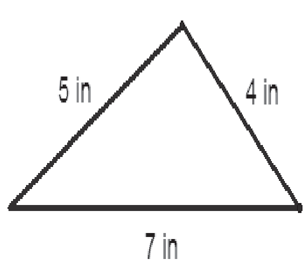
Solution
The perimeter of the rectangle can be found by taking the sum of all four sides.
$P = 4ft + 3ft + 4ft + 3ft = 14ft$

Since the opposite sides of a rectangle are equal the perimeter of a rectangle can also be found by using the formula $P = 2l + 2w$

Let's find the perimeter of the rectangle in example 1 using this formula: $P = 2l + 2w = 2(4ft) + 2(3ft) = 8ft + 6ft = 14ft$

Example 2

Find the perimeter of the following triangle.



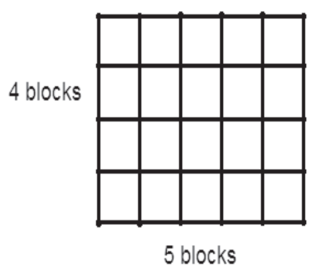
Solution

There isn't a special formula to find the perimeter of a formula. So, simply find the sum of the sides of the triangle.

$P = 5inches + 4inches + 7inches = 16inches$

1.2.2 Understanding Area

The next property of two dimensional objects we will investigate is area. The area of an object is the amount of surface that the object occupies. The area of object depends on its shape. Different shapes use different formulas to compute the area. We will start by finding the area of a rectangle. The area of a rectangle can be found by multiplying the length of the rectangle by the width of the rectangle. Let's examine rectangles further to see why the formula of a rectangle is length times width. Suppose we had a rectangle that was 5 blocks by 4 blocks. This would mean that we would have 4 rows of blocks that each have 5 blocks in them. We could count the number of blocks or find the area by multiply the 4 rows of blocks by the 5 block that are in each rows. Therefore, the formula to find the area of a triangle would be $A = lw$



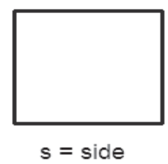
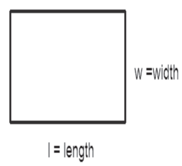
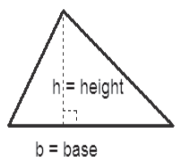
Using the formula $A = lw$, we get that the area of the rectangle is: $Area = (length)(width) = (5units)(4units) = 20$ square units

1.2.3 Computing Areas

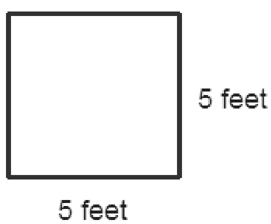
Similarity, formula for the area of other objects can be used to find the area.

Key Formulas

Here are some of the key area formulas that will used in this section.

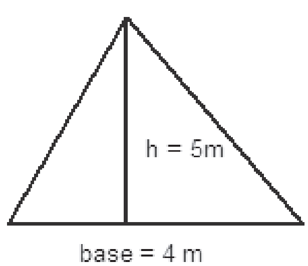
Object	Shape	Formula
Square		$A = s^2$
Rectangle		$A = lw$
Triangle		$A = \frac{1}{2}bh$

Example 3
Find the area of the square.



Solution
To get the answer, substitute the value of the length of the side of square into the area of a square formula.
$A = (5ft)^2 = 25ft^2$

Example 4
Find area of triangle with a base of 4 meters and a height of 6 meters.



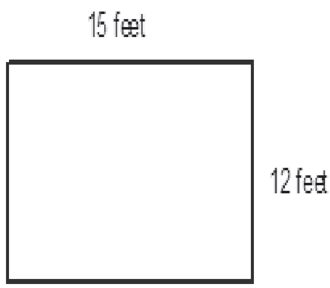
Solution
To get the answer, substitute the values of the length and width of the triangle into the area of a triangle formula.
$A = \frac{1}{2}(5m)(4m) = \frac{1}{2}(20m^2) = 10m^2$

1.2.4 Applications of Area

Area can be used to calculate the square footage of a house or building. This in turn can be used to help calculate the cost of renovations such as installing flooring or carpets. In this next example, we will use the floor plans of one room of a house to calculate the floor area of the house that will allow us to compute the cost to install new carpeting in the room.

Example 5

Suppose you wanted to install new carpet in one room in your house that is 15 feet by 12 feet, how many square feet of carpet would you need?



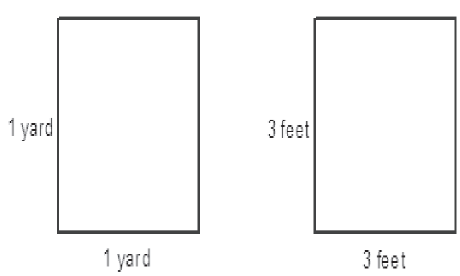
Solution

To get the answer, substitute the values of the length and width of the rectangle into the area of a rectangle formula.
$A = lw = (15in)(12in) = 180in^2$

Now, suppose that price of carpet at the store you are buying carpet from is \$10.50 per square yard. Roughly, how much is it going to cost to put carpet down in the room in Example 1?

Solution
First, find the area of the room in square yards.
$180ft^2(\frac{1yd^2}{9ft^2} = 20yd^2)$
Now, you can find the cost.
$\frac{\$10.50}{ft^2}(20ft^2) = \$210.00$

Note: Why is 1 square yard equal to 9 square feet? If you were to look at a square that is 1 yard by 1 yard, then that square would also be 3 feet by 3 feet. Remember 1 yard is 3 feet.



Therefore, the area of the square in square yards and square feet would be as follows:
$A = (1yard)(1yard) = 1yard^2$ and $A = (3feet)(3feet) = 9feet^2$

Since the two rectangles are the same, 1 square yard = 9 square feet.

Example 6

Suppose you wanted to put down hardwood floors in your living room which is roughly 12 feet by 10 feet. If the cost of the hardwood flooring is $5.00 per square foot, find the approximate cost not including labor to put down hardwood flooring in your living room?

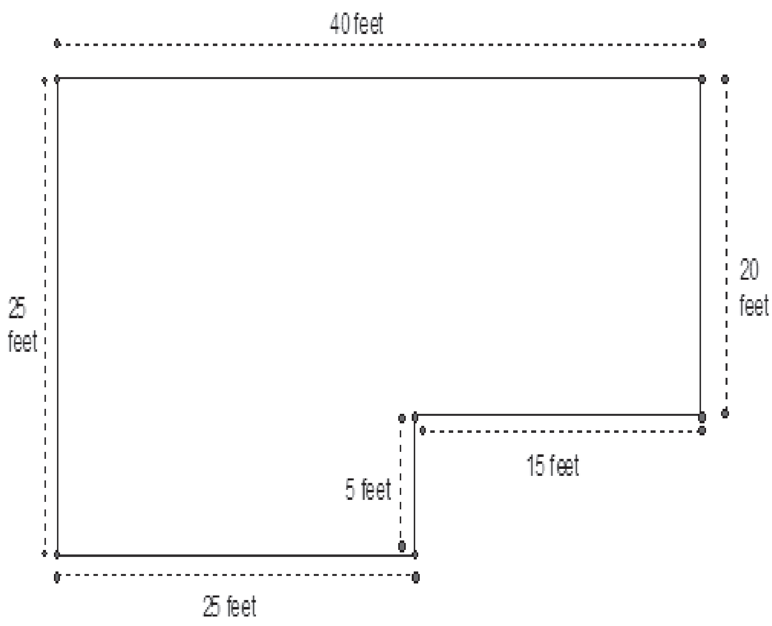
Solution

First, the area of the room, then multiply the area of the rectangle by $5.00 per square foot.

$A = (12ft)(10ft) = 120ft^2$
$Cost = (\frac{\$5.00}{ft^2})(120ft^2)$

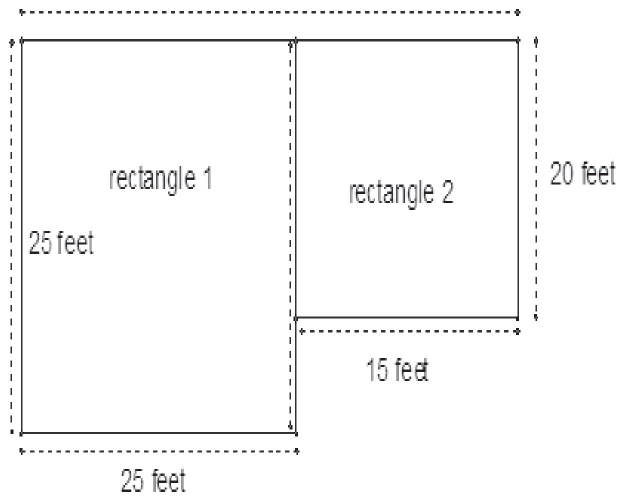
Example 7

Given the floor plans for the 1st floor of a house below, find the area or square footage of the 1st floor of this house.



Solution

Simply divide the rectangle up into two rectangles and find the area of each rectangle

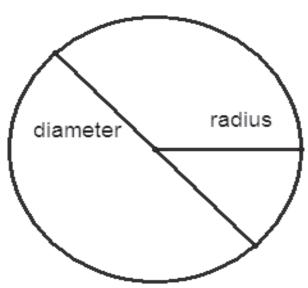


Area of Rectangle 1: $A = (25ft)(25ft) = 625ft^2$
Area of Rectangle 2: $A = (15ft)(20ft) = 300ft^2$

Total Area=Area of Rectangle 1 + Area of Rectangle 2 $= 625ft^2 + 300^2 = 925ft^2$

1.2.5 Circles

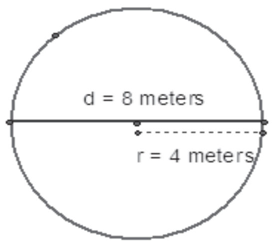
A circle can be defined the set of all points that are equidistance from a point called the center of the circle. Circles are defined by the value of their radius and their center. The radius of a circle is the defined as the straight line distance from the center of the circle to the edge of the circle. The diameter of a circle is a line segment that passes through the center of the circle and has endpoints that lie on the circle



The circumference of a circle is the distance around the circle. The circumference of a circle can thought of as the perimeter of the circle. The circumference of a circle can be found by the following formula: $C = 2\pi r$ or $C = \pi d$ The area of the circle is given by the formula: $A = \pi r^2$ where pi is approximately 3.1416. Pi is defined as the ratio between the circumference of the circle and the diameter of the circle.

Example 8

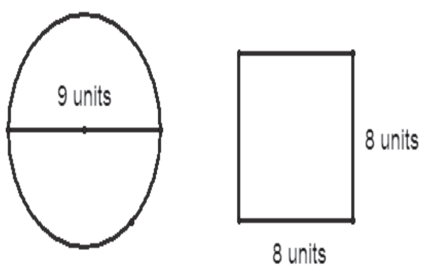
Find the area and circumference of the following circle.



Solution
Use the area of a circle formula to find the area of the circle.
$A = \pi r^2 = \pi(4m)^2 = 16\pi m^2$ or $50.24m^2$
Next, use the circumference formula to find the circumference of the circle.
$C = \pi d = \pi(8m) = 8\pi = 25.12m$

1.2.6 Historical Excursion: Problem 48 of the Rhind Papyrus

The Rhind Papyrus is named after Henry Rhind, a Scottish Historian, who purchased the papyrus in 1858 in Luxor, Egypt. This document can be trace back to around 1650 BC. The Rhind Papyrus is now kept in a British museum along the Egyptian Mathematical Leather Roll which was also purchased by Henry Rhind. One of the famous problems from this document is problem 48. Problem 48 of the Rhind Papyrus translates to the statement "A square of length 8 is the same a circle of 9". We shall investigate this relationship by comparing the area of a square with a side that measures 8 units to area of a circle that has a diameter of 9 units.

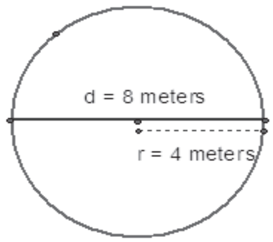


First find the radius of the circle using the fact that the diameter of the circle is 9 units.
$radius = \frac{9units}{2} = \frac{9}{2}$ units

Now, set the area of the circle equal to the area of the square.
$\pi r^2 = (side)(side)$
$\pi(\frac{9}{2})^2 = (8)(8)$
$\pi\frac{81}{4} = 64$
$\pi = (\frac{4}{81})(64)$
$\pi = \frac{256}{81}$

Using this simple calculation you will get an approximate value for pi which is the ratio $\frac{256}{81}$. If divide the value of 256 by 81, you will get a value that is approximately equal to 3.16. Earlier in the chapter we used the modern approximation for pi which is about 3.1416, so as we can see this value of $\frac{256}{8}$ is fairly close to the modern approximation for pi. This ratio of $\frac{256}{8}$ is sometimes refer to as the Egyptian approximation for Pi. Let's used this Egyptian approximation for Pi to calculate the area of a circle.

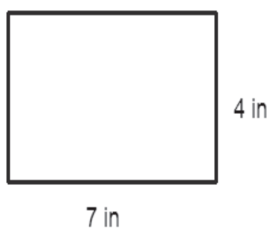
Example 9
Use the Egyptian Approximation for π, $\frac{256}{81}$, to find the area and circumference in problem 8.



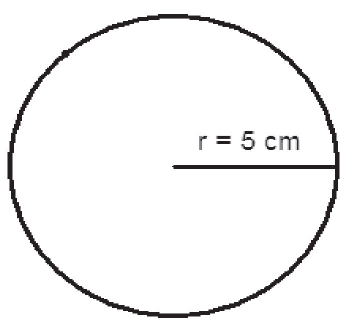
Solution
Use the area of a circle formula to find the area of the circle.
$A = \pi r^2 = \pi(4m)^2 = 16\pi m^2 = 16(\frac{256}{81}) = \frac{4096}{81} = 50.57m^2$
Next, use the circumference formula to find the circumference of the circle.
$C = \pi d = \pi(8m) = 8\pi = 8(\frac{256}{81}) = \frac{2848}{81} = 35.16$ m

1.2.7 Exercises

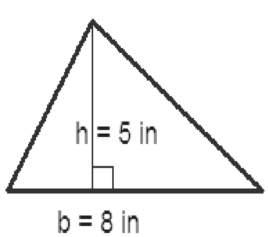
1. Find the perimeter and area of the following rectangle.



2. Find the circumference and area of the following circle.

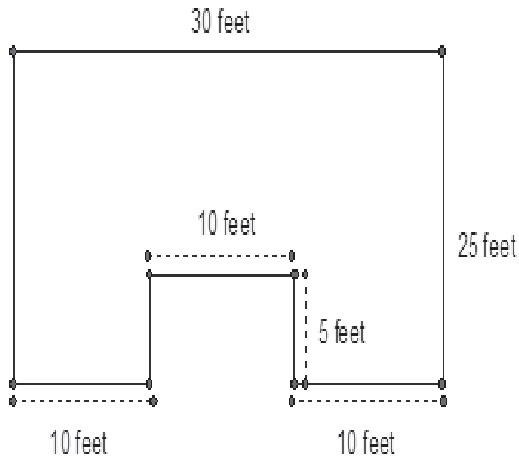


3. Find the area of the following triangle.



4. Find the area of a rectangular shaped lot with a length of 40 yards and a width of 60 yards.
5. Find the area and perimeter of a square shaped lot with a length of 100 meters.
6. You want to put down hardwood floors in your living room that is roughly 18 feet by 16 feet. If the cost of the hardwood flooring is $5.00 per square foot, find the approximate cost not including labor to put down hardwood flooring in your living room?
7. You want to put down carpet in your living room that is roughly 6 yards by 2 and half yards. If the cost of the carpet is $4.00 per square yard, find the approximate cost not including labor to put down hardwood flooring in your living room?

8. Given the floor plans for the 1st floor of a house below, find the area or square footage on the 1st floor of this house.



9. 1 $ft^2 = ?\ in^2$

10. 1 $yd^2 = ?\ ft^2$

11. Use the Egyptian value for Pi to find the area of a circle with a radius of 4 centimeters. Then, use the value of 3.14 for Pi to find the area of the circle. Compare the two values.

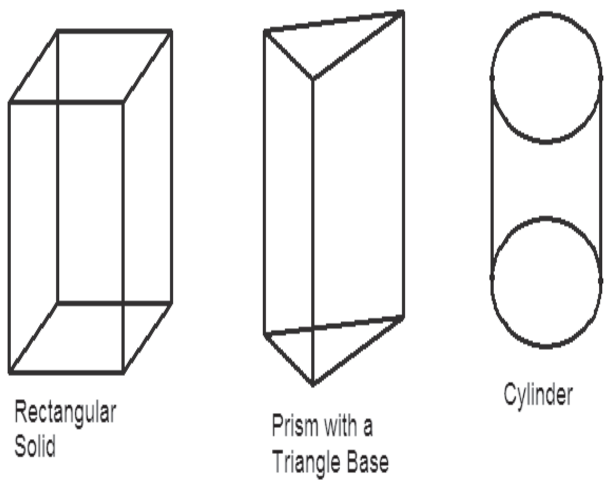
1.3 Volume

1.3.1 Understanding Volume

The main topic of this section is volume. You will specifically look at how to find the volume of various three dimension geometric objects such as rectangular solids and cylinders. The volume of an object is the amount space occupied by the object. We will use formulas to find the volume of an object in the same way we used formulas to find the area of an object. Let's begin by classifying three dimensional objects into two categories. The two categories are prism and pyramids. A prisms is a three dimension object that has two bases that are identical and a pyramid is a three dimensional object with just one base.

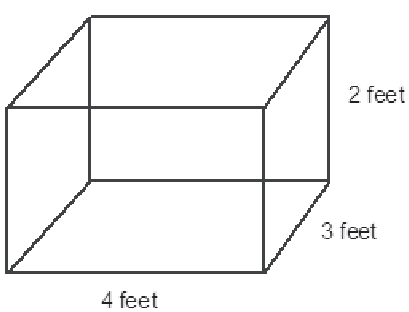
1.3.2 Prisms

In the next diagram there are pictures of a rectangular solid, a prism with a triangular base, and a cylinder. These are just a few of the many possibilities that we have for objects that we classify as prisms. Notice that in each example, the objects have a base at the top and the bottom are identical.



To find the area of any prism, we simply multiply the base of the prism by the height of the prism. $Volume = (Base)(Height)$ In next few examples, we will investigate how to find the volume of a prism such as a rectangular solid or cylinder.

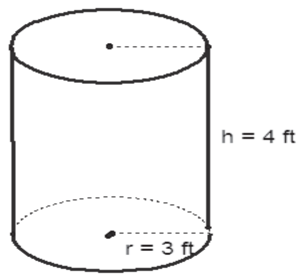
Example 1: Rectangular Solids
Find the following of the following rectangular solid.



Solution

Since the object is a prism, we will use the formula for the volume of a prism which is $V = Bh$. The base of the rectangular solid is a rectangle. Therefore, the formula to find the volume would be $V = Bh = (lw)h = lwh$
$V = (4ft)(3ft)(2ft) = (12ft^2)(2ft) = 24ft^3$

Example 2: Cylinders
Find the volume of a cylinder with a radius of 3 inches and height of 4 inches.

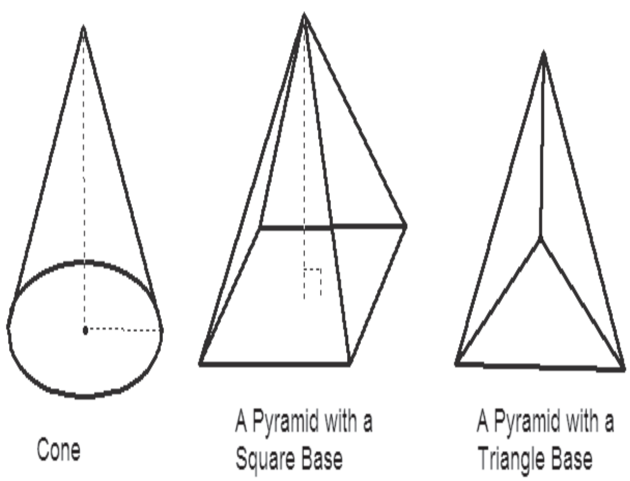


A cylinder is a prism with a circular base. Using the formula for the prism along with the formula of the area a circle, we get the following formula to find the area of a cylinder. $V = Bh = (\pi r^2)h = \pi r^2 h$

$V = \pi(3in)^2(4in) = \pi(9in^2)(4in) = 36\pi in^3$
Using the value of 3.14 for Pi we would get $V = 36\pi in^3 = 113.04in^3$

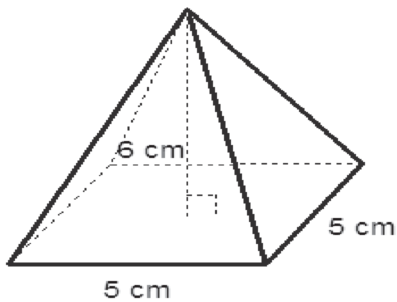
1.3.3 Pyramids

Pyramids like prism can have number of different shape provided the object has only one base. Here is a diagram that illustrates a few of the types of pyramids that exist.



Volume of Pyramids To find the volume of a pyramid, we multiply one third times the base multiplied by the height. The formula to compute the volume of a pyramid is $V = \frac{1}{3}Bh$ In next few examples, we will investigate how to find the volume of some pyramid.

Example 3
Find the area of the following pyramid with a square base



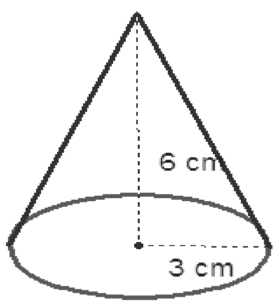
Solution

Since the object is a pyramid with a square base, we can find the volume by computing $\frac{1}{3}$ the area of the square base and multiplying by the height. $V = \frac{1}{3}Bh = \frac{1}{3}(lw)h = \frac{1}{3}lwh$

$V = \frac{1}{3}(5cm)(5cm)(6cm) = \frac{1}{3}(150cm^3) = 50cm^3$

Example 4

Find the volume.



Solution

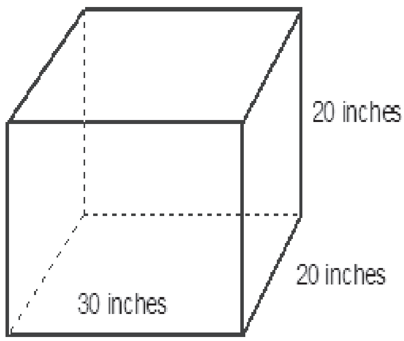
A cylinder is a form of a pyramid, we can use the formula for the volume of a pyramid. $V = \frac{1}{3}Bh$ Since the base of a cylinder is a circle, we will use the area of a circle for the base.
$V = \frac{1}{3}(\pi r^2)h$
$V = \frac{1}{3}(\pi(3cm)^2)(6cm)$
$V = \frac{1}{3}(9\pi cm^2)(6cm)$
$V = (3\pi cm^2)(6cm)$
$V = 18\pi cm^3$

1.3.4 Applications of Volume

We use volumetric measurement all the time. We pump are gasoline from the service station in gallons. We buy our beverages from the grocery store in liters and pints. Our water bills are computed according to the gallons of water that we use. Let's look at some examples where we would calculate the volume of common objects in different units of measure.

Example 5

A fish aquarium shaped like a rectangular solid is 30 inches wide, 20 inches long, and 20 inches tall. How much volume could the fish aquarium hold in both cubic inches?



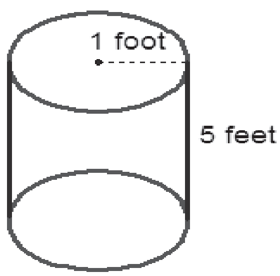
Solution

Since the fish aquarium is a rectangular solid, just substitute the length, width, and height of the aquarium into formula of the volume of a rectangular solid. The final units will be in cubic inches.

$V = (30cm)(20cm)(20cm) = (600cm^2)(20cm) = 12000cm^3$

Example 6

A hot water heater is shaped like a cylinder that has a radius of 1 foot and a height of 5 feet. How much water can the hot water heater hold in cubic feet and gallons? Hint: 1 foot = .134 gallons



Solution

Use the volume of cylinder to find the volume of water in the hot water heater.

$V = \pi r^2 h = \pi(1ft)^2(5) = \pi(1ft^2)(5ft) = 5\pi ft^3 = 15.7ft^3$

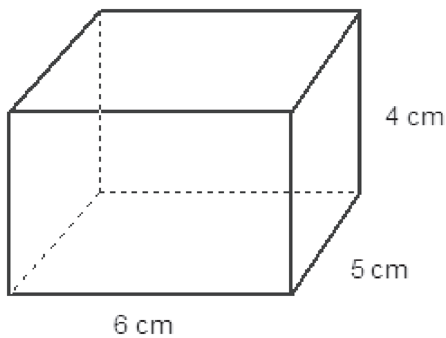
Next, convert the cubic feet to gallons.

$15.7ft^3(\frac{1gallon}{.134ft^3}) = 117.2gallons$

1.3.5 Surface Area

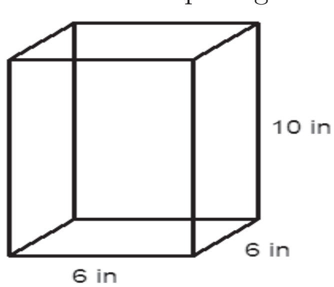
Every three dimensional object has surface area as well volume. The volume, as stated earlier, measured the amount of space occupied by a three dimension object. The surface area of a three dimension object measures amount of surface of the object. The surface area of object such as a cube, rectangular solid, pyramid, or cylinder is found by finding the area of each face and then finding the sum of the faces. For example, suppose you had to find the surface of a rectangular solid. You would have to find the area of top face, bottom face, front face, the face in the back, and the two sides. Let's try an example.

Example 7
Find the area of a rectangular that has a length of 6 feet, height of 4 feet, and a width of 5 feet.



Solution
To find the surface area of the object, you find the area of each face and then sum up all of the faces. The faces of the rectangular solid are all rectangles. Therefore, you can use the formula for the area of a rectangle to find the area of each face. If you look at the drawing of the rectangular solid, you can tell that the front and back are the same area. Likewise, the top and bottom are the same area and the two sides are the same area.
Area of top and bottom: $A = lw = (6cm)(5cm) = 30cm^2$
Area of front and back: $A = lw = (6cm)(4cm) = 24cm^2$
Area of two sides: $A = lw = (5cm)(4cm) = 20cm^2$
Total Area $= 30cm^2 + 24cm^2 + 20cm^2 = 74cm^2$

Example 8
Suppose you want to mail a rectangular solid shaped package that measures 6 inches by 6 inches by 10 inches. How much postal wrap do you need to completely cover the package? Provided that the package weighs less than 20 pounds, the postal rate is 2.5 cents per cubic inch. Find the postage charge on the package.

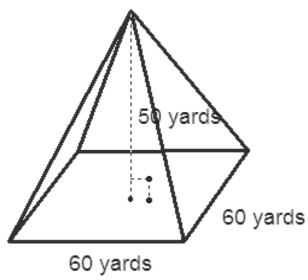


Solution
First find the surface area by adding up all the the faces of the package.
Area of top and bottom: $A = lw = (6in)(6in) = 36in^2$
Area of front and back: $A = lw = (6in)(10in) = 60in^2$
Area of two sides: $A = lw = (6in)(10in) = 60in^2$
Total Area $= 36in^2 + 60in^2 + 60in^2 = 220in^2$

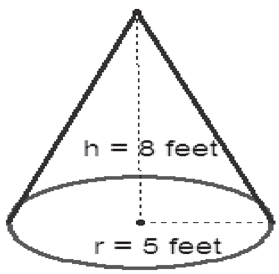
If the cost to mail the package is 2.5 cents per cubic inch, then multiply the volume of the package by the postal rate. The volume of the package is $V = (6in)(6in)(10in) = 360in^3$
Therefore, the postal charge would be $(360in^2)(\frac{\$.025}{1in^3}) = \$9.00$

1.3.6 Exercises

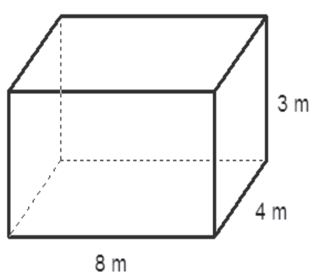
1. Find the volume of a cylinder with a radius of 3 inches and height of 4 inches.
2. Find the volume of the following pyramid with a square base.



3. Find the volume of the following cone.



4. Find the volume of a cone with a radius of 6 centimeters and a height of 8 centimeters
5. Suppose you have a cylinder shaped hot water heater that has a height of 5 feet and a radius of 1 foot. How water can the hot water heater hold in cubic feet and gallons?
6. Find the volume of following rectangular solid.



7. A fish aquarium shaped like a rectangular solid is 30 inches wide, 20 inches long, and 20 inches tall. How much volume could the fish aquarium hold?
8. A fish aquarium shaped like a rectangular solid is 20 inches wide, 25 inches long, and 15 inches tall. How much volume could the fish aquarium hold in cubic inches and gallons?
9. Suppose you want to mail a rectangular solid shaped package that measures 10 cm by 12 cm by 12 cm. How much postal wrap do you need to completely cover the package?
10. Find the surface area of a box that is 2 feet by 3 feet by 4 feet.

1.4 Similar Triangles

1.4.1 Similarity

If two objects have the same shape but have different size, they are called similar objects. For example a model boat might have the same size as the original boat, but a different size than the original boat. Model trains are also scaled to a fraction amount of the actual sized train. This fraction amount is called a called a scaling ratio. HO model train are $frac187$ the size of the original train and N scale trains are $\frac{1}{160}$ the size of the original train.

Before solve a problem using ratios let's review how to solve a proportion.
Example 1

Solve the following proportion. $\frac{x}{10} = \frac{18}{24}$

To solve first take the cross product of the proportion:
$\frac{x}{10} = \frac{18}{24}$
$24x = (10)(18)$
Now, just solve the proportion for x. $24x = 180$
$\frac{24x}{24} = \frac{180}{24}$
$x = 7.5$
Note that the solution does check: $\frac{7.5}{10} = .75$ and $\frac{18}{24} = .75$

Example 2

Solve the following proportion. $\frac{4}{x+2} = \frac{11}{22}$

To solve first take the cross product of the proportion:
$\frac{4}{x+2} = \frac{11}{22}$
$11(x + 2) = (4)(22)$
Now, just solve the proportion for x by subtracting 22 from both sides of the equation and

dividing by 11. $11x + 22 = 88$

$11x + 22 - 22 = 88 - 22$
$11x = 66$
$x = 6$

Example 3
A HO scale model train is 8 inches long. How long would the original train be in feet?

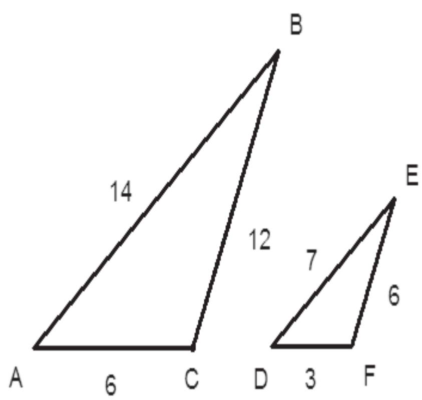
Solution
To find the length of the real sized train, simply set up an ratio us the HO scale which is $\frac{1}{87}$.

$\frac{1}{87} = \frac{8inches}{x}$
$1(x) = 87(8in)$
$x = 696inches$

Length of the original train $= 696in(\frac{1ft}{12in}) = 58feet$

1.4.2 Similarity Triangles

Two Triangles are similar if the measures of their corresponding angles are congruent and their corresponding sides are proportional. Below is an example of two triangles that are similar.



In the diagram above the sides are proportional:
$\frac{AC}{DF} = \frac{6}{3} = 2$

$\frac{CB}{FE} = \frac{12}{6} = 2$

$\frac{AB}{DE} = \frac{14}{7} = 2$

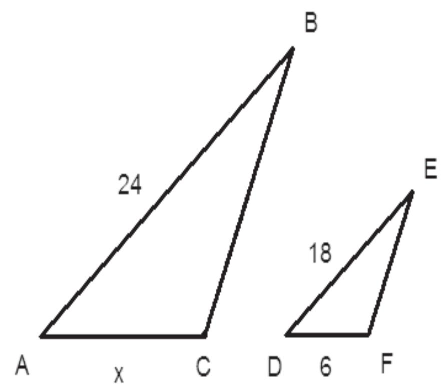
The angles are also congruent.
$\angle A \cong \angle D$, $\angle \cong \angle E$, and $\angle C \cong \angle F$

Note: we would write triangle ABC is similar to triangle DEF as follows $\triangle ABC \sim \triangle DEF$

Now let's use the properties of congruent triangle to help us find the missing side of a triangle.

1.4.3 Applications of Similar Triangles

Example 4
Find the value of x given that $\triangle ABC \sim \triangle DEF$

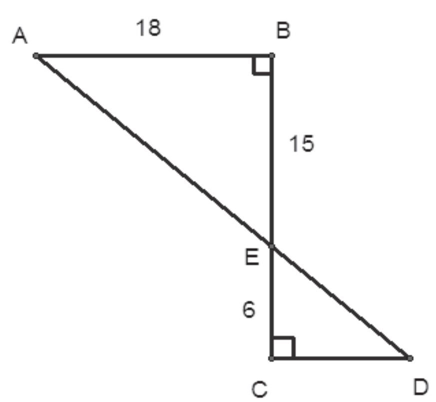


Solution

Use the relationship between the sides of the two triangle to set up a proportion and solve for x.
$\frac{24}{18} = \frac{x}{6}$
$18(x) = 24(6)$
$18x = 144$
$x = 8$

Example 5

Find the value of CD given that AB is parallel to CD.

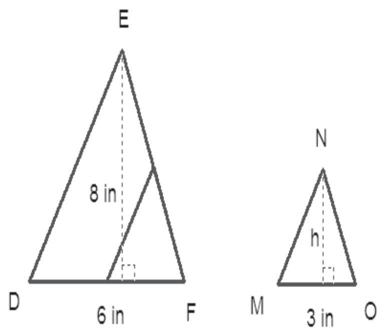


Solution Since AB is parallel to CD, $\angle A$ and $\angle D$ are equal. $\angle B$ and $\angle C$ are right angle, so these angles are also congruent. This makes the third pair of angles congruent. Therefore,

the $\triangle ABE$ similar to $\triangle CDE$. Now, we can set up a proportion between the sides of the triangle.
$\frac{AB}{BE} = \frac{CD}{DE}$
$\frac{18}{15} = \frac{CD}{6}$
$15CD = (18)(6)$
$15CD = 144$
$CD = 9.6$

Example 6

Triangles DEF and MNO are similar triangles, find the area of MNO.

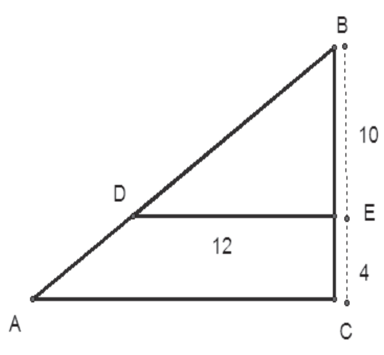


to find the area of triangle MNO, we need to find the height of MNO. We can find the height of MNO by setting up a proportion.

$\frac{h}{3} = \frac{8}{6}$
$6h = (3)(8)$
$6h = 24$
$h = 4in$

After finding the height of the triangle, we can substitute the base and height of the triangle into the area of a triangle formula to find the area
$Area = \frac{1}{2}bh = \frac{1}{2}(3in)(4in) = 6in^2$

Example 7
Find the value of AC given that $\triangle ABC \sim \triangle DBE$



Solution
First find the value of BC which is $BC = BE + EC = 10 + 4 = 14$
Then use a proportion to find AC

$\frac{DE}{AC} = \frac{BE}{BC}$
$\frac{12}{AC} = \frac{10}{14}$
$10AC = (12)(14)$
$10AC = 168$
$CD = 16.8$

1.4.4 Exercises

1. Solve the following proportion. $\frac{x}{6} = \frac{12}{18}$

2. Find the value of x. $\frac{6}{x+2} = \frac{12}{22}$

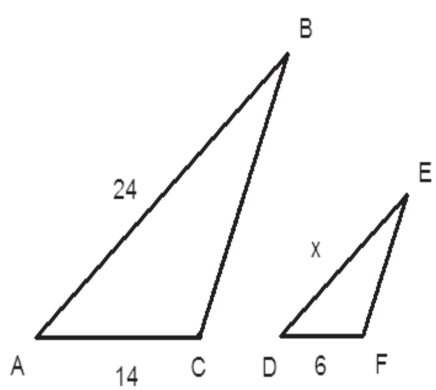
3. Neil's new hybrid car gets 56 miles per gallon on the open highway. Rick's hybrid car can go 440 miles on 9 gallons on the open highway. Who has a more fuel efficient car?

4. The British Pound is worth one dollar and 70 cents in American currency. Using this conversion rate, 10 U S dollars would exchange to how many British Pounds?

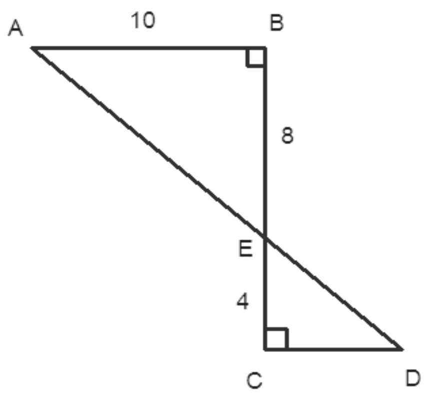
5. John is 6 feet tall and casts a shadow of 16 feet at 3 pm today. At the same time, an maple tree in John's yard casts a 55-foot shadow. How tall is John's maple tree.

6. A certain shade of purple can be made by mixing 4 drops of blue paint with 5 drops of red paint. Using this ratio how many drops of blue paint must be mixed with 50 drops of red paint.

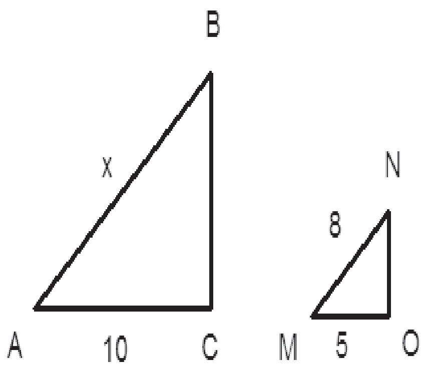
7. Find the value of x given that $\triangle ABC \sim \triangle DEF$



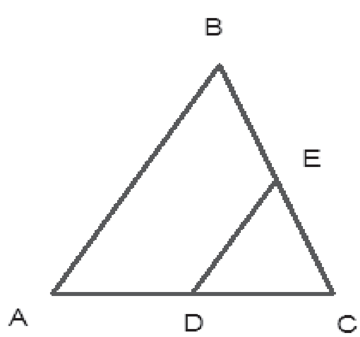
8. Find the value of CD given that AB is parallel to CD.



9. Find the value of x given that $\triangle ABC \sim \triangle MNO$



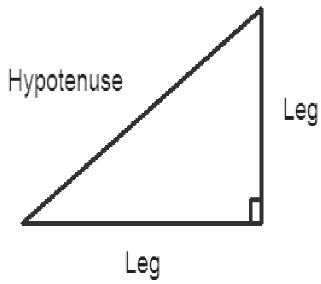
10. Find the value of BC given that $\triangle ABC \sim \triangle DEC$ and $AB = 25in$, $DE = 15in$, and $EC = 6in$



1.5 Pythagorean Theorem

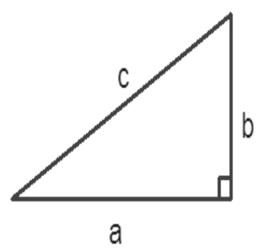
1.5.1 Right Triangles

Before we begin to study the Pythagorean Theorem, let's discuss some facts about right triangles. The longest side of a right triangle which opposite the right angle is called the hypotenuse. The other two sides that formed the right angle are called the legs.



In every right triangle, there is the same relationship between the legs of the right triangle and the hypotenuse of the right triangle. This relationship is know as the the Pythagorean Theorem.

In a right triangle, the sum of the squares of the two legs is equal to the square of the



hypotenuse. $c^2 = a^2 + b^2$

1.5.2 Find the missing side of a right triangle

Example 1
Find the length of the missing side: a = 9, b = 12, c = ?

solution
Simple substitute the value of a and b into the Pythagorean Theorem and solve for c.
$c^2 = a^2 + b^2$
$c^2 = 9^2 + 12^2$
$c^2 = 81 + 144$
$c^2 = 225$
$\sqrt{c^2} = \sqrt{225}$
$c = 15$

Example 2
Suppose the two legs of a right triangle are 5 units and 12 units, find the length of the hypotenuse.

SolutionTo find the solution, substitute the value of the legs into the Pythagorean Theorem

and solve for the hypotenuse.
$c^2 = a^2 + b^2$
$c^2 = 5^2 + 12^2$
$c^2 = 25 + 144$
$c^2 = 169$
$\sqrt{c^2} = \sqrt{169}$
$c = 13$

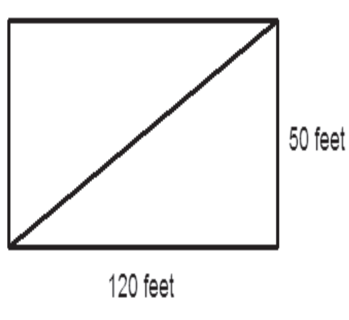
Example 3
Suppose that the hypotenuse of a right triangle is 26 units and one leg is 10 units, find the measure of the other leg.

Solution
To find the answer, substitute the value of the leg and hypotenuse into the Pythagorean Theorem and solve for the missing leg.
$c^2 = a^2 + b^2$
$26^2 = 10^2 + b^2$
$676 = 100 + b^2$
$b^2 = 576$
$\sqrt{b^2} = \sqrt{576}$
$c = 24$

1.5.3 Applications of the Pythagorean Theorem

The Pythagorean Theorem has several real life applications. This is due to the fact that so many problems can be modeled or represented by a right triangle. If this is the case, then values can be assigned to the sides of the triangle and the unknown value can be found by solving for the missing side of the triangle. Here are some examples of applications of right triangles and the Pythagorean Theorem

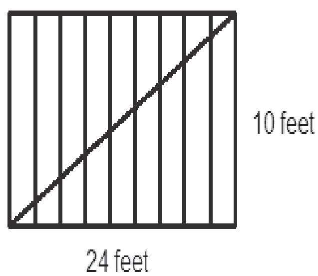
Example 4
An empty lot is 120 ft by 50 ft. How many feet would you save walking diagonally across the lot instead of walking length and width?



Solution
To find the answer, substitute the value of the legs into the Pythagorean Theorem and solve for the hypotenuse.
$c^2 = a^2 + b^2$

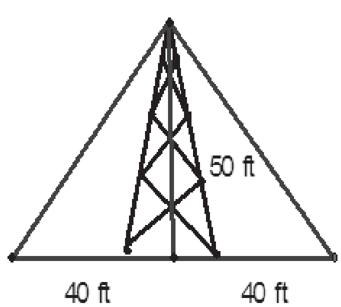
$c^2 = 50^2 + 120^2$
$c^2 = 2500 + 14400$
$c^2 = 16900$
$\sqrt{c^2} = \sqrt{16900}$
$c = 130$
Often construction workers and carpenters while use angles and triangle in their profession. For example, carpenters can use the Pythagorean Theorem to find measurements when constructing the walls of a house. In this case, we will find the length of a brace that is put inside the walls of the house to reinforce the walls.

Example 5
A diagonal brace is to be placed in the wall of a room. The height of the wall is 10 feet and the wall is 24 feet long. (See diagram below) What is the length of the brace?



Solution
To find the answer, substitute the value of the legs into the Pythagorean Theorem and solve for the hypotenuse.
$c^2 = a^2 + b^2$
$c^2 = 10^2 + 24^2$
$c^2 = 100 + 576$
$c^2 = 676$
$\sqrt{c^2} = \sqrt{676}$
$c = 26 feet$

Example 6
A television antenna is to be erected and held by guy wires. If the guy wires are 40 ft from the base of the antenna and the antenna is 50 ft high, what is the length of each guy wire?

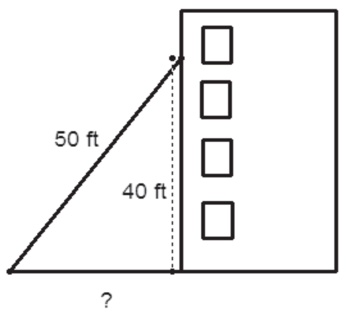


Solution Substitute into the Pythagorean Theorem using the values 40 feet and 50 feet in for the legs of the right triangle, and solve for the hypotenuse.

$c^2 = a^2 + b^2$
$c^2 = 40^2 + 50^2$
$c^2 = 1600 + 2500$
$c^2 = 4100$
$\sqrt{c^2} = \sqrt{4100}$
$c = 64 feet$

Example 7

Given that a 50 foot ladder rest against a window ledge that is 40 feet above the ground, find out how far is the ladder from the edge the building?



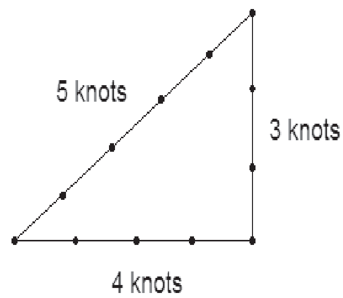
Solution

Use the Pythagorean theorem to find the distance the ladder is from the building by using the length of the ladder as the hypotenuse and height of the building as the legs of the triangle. After substituting into the Pythagorean theorem, solve for the other leg.

$c^2 = a^2 + b^2$
$50^2 = 40^2 + b^2$
$2500 = 1600 + b^2$
$b^2 = 900$
$\sqrt{b^2} = \sqrt{900}$
$b = 30 feet$

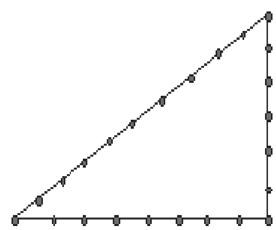
1.5.4 Historical Excursion: The Pythagorean Theorem

The origins of right triangle geometry can be traced back to 3000 BC in Ancient Egypt. The Egyptians used special right triangles to survey land by measuring out 3-4-5 right triangles to make right angles. The Egyptians mostly understood right triangles in terms of ratios or what would now be referred to as Pythagorean Triples. The Egyptians also had not developed a formula for the relationship between the sides of a right triangle. At this time in history, it is important to know that the Egyptians also had not developed the concept of a variable. The Egyptians most studied specific examples of right triangles. For example, the Egyptians use ropes to measure out distances to form right triangles that were in whole number ratios. In the next illustration, it is demonstrated how a 3-4-5 right triangle can be form using ropes to create a right angle.

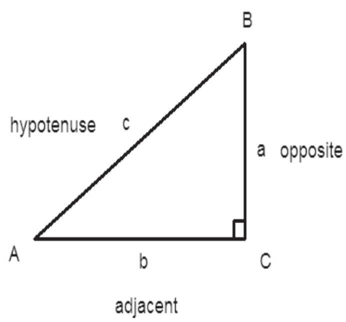


Using ropes that had knots that where equality spaced, the Egyptians could measure out right angles by making a 3-4-5 right angle or other right triangles with the rope.
It wasn't until around 500 BC, when a Greek mathematician name Pythagoras discovered that there was a formula that described the relationship between the sides of a right triangle. This formula was known as the Pythagorean Theorem.

Example 8
Determine if the triangle measured out by ropes has a right angle.



If you count the number of knots on each side of the triangle you get a ratio of 6-8-10.



Substituting these values into the Pythagorean Theorem using 10 as the hypotenuse and the other two sides as the legs, you can determine if the triangle is a right triangle.
$c^2 = a^2 + b^2$
$10^2 = 6^2 + 8^2$
$100 = 36 + 64$
$100 = 100$

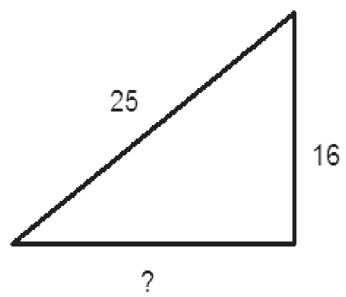
After substituting these values into the Pythagorean Theorem, the final values obtained on both sides of the equation are equal. This verifies that the triangle is a right triangle. Therefore, the triangle has a right angle.

1.5.5 Exercises

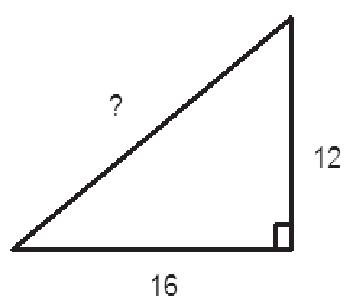
1. Find the length of the missing side: a = 6, b = 8, and c = ?

2. Find the length of the missing side. a = 12, c = 20, b = ?

3. Find the length of the missing leg in diagram below: (Round answer to the nearest tenth)

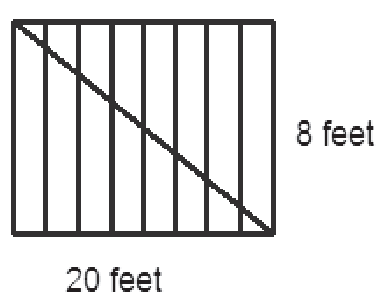


4. Find the length of the missing side.



5. A rectangular shaped lot is 80 ft by 60 ft. How many feet would you save walking diagonally across the lot instead of walking length and width?

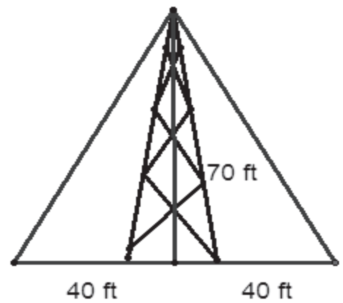
6. A diagonal brace is to be placed in the wall of a room. The height of the wall is 8 feet and the wall is 20 feet long. (See diagram below) What is the length of the brace?



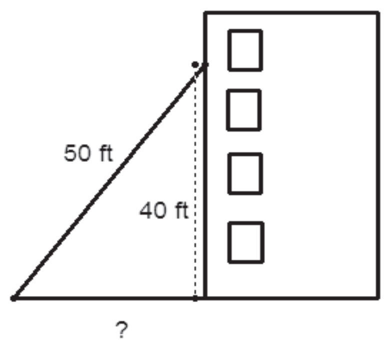
7. Find the length diagonal of a rectangle that is 30 ft by 40 ft.

8. Find the length diagonal of a rectangle that is 30 ft by 30 ft.

9. A television antenna is to be erected and held by guy wires. If the guy wires are 40 ft from the base of the antenna and the antenna is 70 ft high, what is the length of the guy wire?



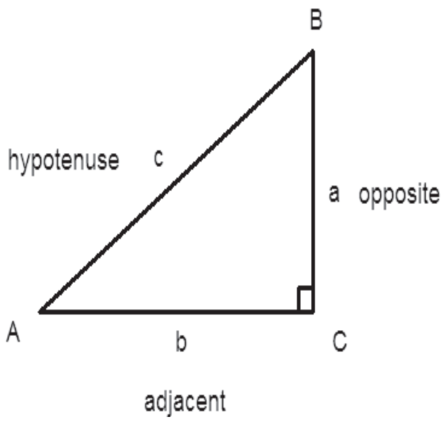
10. Given that 35 foot latter rest against a window ledge that is 25 feet above the ground, find out how far is the latter form the edge the building?



1.6 Introduction to Trigonometry

1.6.1 The Basic Trigonometric Ratios

The basic trigonometric ratios are derived from a basic right triangle where the two legs are labeled as opposite and adjacent and the longest side is labeled as the hypotenuse. The three trigonometric ratios we will study in this course are sine of the angle, cosine of the angle, and tangent of the angle. If we are measuring these ratios with respect to the angle A, they would be abbreviated as sin A, cos A, and tan A. In the next figure, the side opposite angle A is side a, the side adjacent to A is b, and the hypotenuse is c.



The trigonometric ratios sin A, cos A, and tan A are defined as the following ratios.

$$\sin A = \frac{opposite}{hypotenuse}$$
$$\cos A = \frac{adjacent}{hypotenuse}$$
$$\tan A = \frac{opposite}{adjacent}$$

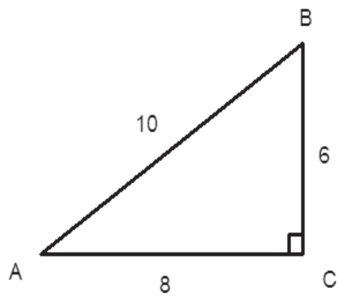
The other three trigonometric functions which we will not use in this course are secant (sec), cosecant (csc), and cotangent (cot).

$$\sec A = \frac{hypontenuse}{opposite}$$
$$\cos A = \frac{hypotenuse}{adjacent}$$
$$\cot A = \frac{adjacent}{opposite}$$

Here are a few examples how to find the trigonometric ratios for a given triangle

Example 1

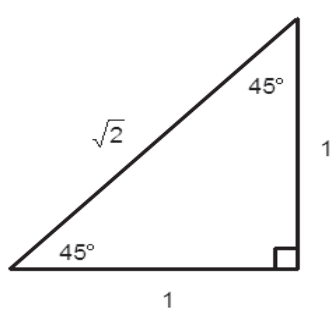
Use the following triangle to find the ratios sin A, cos A, and tan A, then use the same triangle to find the ratios sin B, cos B, and tan B.



Solution
First find the values of sine, cosine, and tangent in terms of angle A (sin A, cos A, tan A)
$\sin A = \frac{opposite}{hypotenuse} = \frac{6}{10} = \frac{3}{5}$
$\cos A = \frac{adjacent}{hypotenuse} = \frac{8}{10} = \frac{4}{5}$
$\tan A = \frac{opposite}{adjacent} = \frac{6}{8} = \frac{3}{4}$

Next, find the values of sine, cosine, and tangent in terms of angle B
$\sin B = \frac{opposite}{hypotenuse} = \frac{8}{10} = \frac{4}{5}$
$\cos B = \frac{adjacent}{hypotenuse} = \frac{6}{10} = \frac{3}{5}$
$\tan B = \frac{opposite}{adjacent} = \frac{8}{6} = \frac{4}{3}$

Example 2
Use the following right triangle to find the value of $sin(45^0)$,$cos(45^0)$, and $tan(45^0)$



Solution
Use the three formulas for trigonometric function along with the values of $\sqrt{2}$ for the hypotenuse and 1 for the legs to get the following ratios.
$\sin(45^0) = \frac{opposite}{hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = .707$
$\cos(45^0) = \frac{adjacent}{hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = .707$
$\tan(45^0) = \frac{opposite}{adjacent} = \frac{1}{1} = 1$

1.6.2 Using a Calculator to Find the Value of the Trigonometric Functions

Next, we will use a standard scientific calculator to find value of the trig functions. Most scientific calculators will special keys for the sine, cosine, and tangent functions. In this first example we will find the trigonometric values of a given angle measure.

Example 3

Use a calculator to find the value of trigonometric functions in Example 2. ($sin(45^0)$,$cos(45^0)$, and $tan(45^0)$) Round answers to the nearest thousandth.
$sin(45^0) = .707$
$ocs(45^0) = .707$
$tan(45^0) = 1.000$

Notice that these answer match the answer from Example 2

Example 4

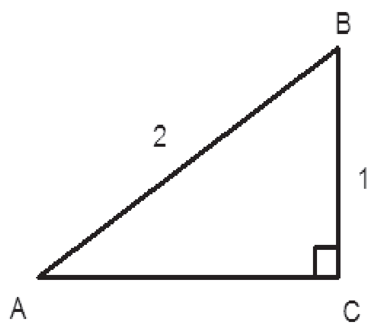
Use a scientific calculator to find each value.
a) $\sin(50^0)$ b) $\cos(50^0)$ c) $\tan(80^0)$ d) $\cos(0^0)$

Solutions

Using your scientific calculator, see if you get the same answers as below:
$\sin(50^0) = .766$
$\cos(50^0) = .642$
$\tan(80^0) = 5.67$
$\cos(0^0 = 1.000)$

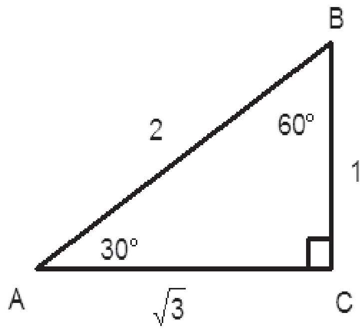
Inverse Trigonometric Functions

Suppose that you didn't know the did know the value of the angle in a triangle, but you did know that the sine of the angle measure was equal to one half. If we were to assigned the variable A to the missing angle, then this would result in the following equation. $\sin A = \frac{1}{2}$ To solve this equation, you would then have to rewrite the equation as an inverse sine function. $A = \sin^{-1}(\frac{1}{2})$ Recall that if $\sin A = \frac{1}{2}$, then the hypotenuse of the triangle would be 2 and the side opposite the angle A would be 1. Here is a picture of the triangle with a hypotenuse that is 2 units and opposite side is 1 unit.



Using the Pythagorean Theorem, you can find the missing side of the right triangle
$c^2 = a^2 + b^2$
$2^2 = 1^2 + b^2$
$4 = 1 + b^2$
$b^2 = 3$
$\sqrt{b^2} = \sqrt{3}$
$b = \sqrt{3}$

The ratio of the sides of the triangle is $1 : \sqrt{3} : 2$. This results in a $30^0 - 60^0 - 90^0$ triangle.



From the drawing of the triangle, we can see that angle A is 30^0
At this point you can also use the inverse sine key on your calculator, which would give you the following result: $A = sin^{-1}(\frac{1}{2}) = 30^0$

Inverse Trigonometric Functions and Calculators

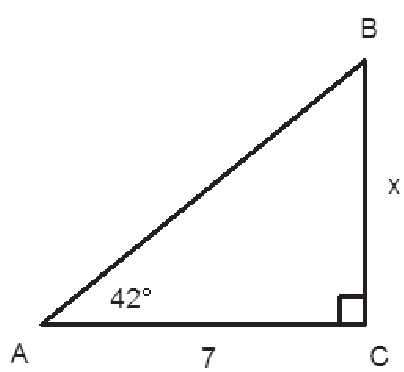
Example 4
Use a calculator to find the value of each inverse function
a) $cos^{-1}(\frac{1}{2})$ b) $cos^{-1}(.345)$ c) $tan^{-1}(\frac{1}{3})$

Solution
Using the inverse function key along with the correct trig function key on your calculator, you should get the following results.
$cos^{-1}(\frac{1}{2}) = 60^0$
$cos^{-1}(.345) = 69.8^0$
$tan^{-1}(\frac{1}{3}) = 18.4^0$

1.6.3 Using Trigonometry to Find The Missing Side of a Triangle

Right triangle trigonometry can be used to solve for missing sides of a triangle much like the Pythagorean Theorem can be used to solve for missing side of right triangle. However, Trigonometry allows us to find the missing side of a right triangle if we are given only one side of the right and one of the acute angles.

Exercise 5
Find the value of the missing side.



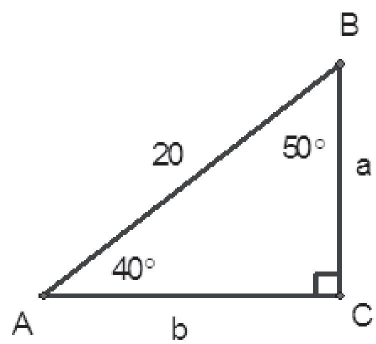
Solution

Since we know the value of the side adjacent to the given angle and we are trying to find the value of the side opposite the angle, we can use the tangent function to solve for the missing side.

$\tan(42^0) = \frac{x}{7}$
$.900 = \frac{x}{7}$
$7(.900) = 7(\frac{x}{7})$
$x = 6.3$

Example 6

Use trigonometry to find the missing sides a and b of the triangle. (Round answer to the nearest tenth)



Solution

Since we know the value of hypotenuse and the angle opposite side A, we can use the sine function to find the value of side a. (Round to the nearest tenth)

$\sin(40^0) = \frac{a}{20}$
$.642 = \frac{a}{20}$
$20(.642) = 20(\frac{a}{20})$
$a = 12.9$

Since we know the value of hypotenuse and the angle adjacent side A, we can use the cosine function to find the value of side b. (Round to the nearest tenth)

$\cos(40^0) = \frac{b}{20}$
$.766 = \frac{b}{20}$
$20(.766) = 20(\frac{b}{20})$
$b = 15.3$

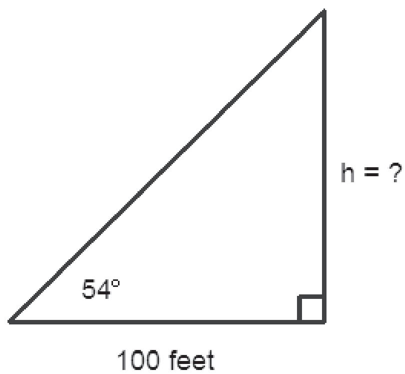
1.6.4 Applications of Trigonometry

Trigonometry can be use to find missing sides and angles of a triangle. If the situation or problem can be modeled by a right triangle, trigonometry can be use to find the solution. For this reason, right triangle trigonometry can be use to find missing horizontal distances, height, angles of depression, angles of elevation, and other values. In this first situation, we are going to use trigonometry to find the height of a cliff.

1.6.5 Angle of Elevation

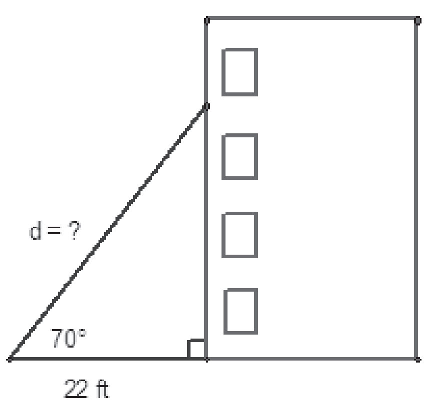
When angle in the problem starts from the ground or line of sight and moves upward, the angle is considered to be the angle of elevation. Here is an example that has an angle of elevation and requires trigonometry to solve it.

Example 7
From a point 100 ft from the base of a cliff, the top of a cliff is seen through an angle of elevation of 54. How tall is the cliff? (Round answer to the nearest tenth)



Solution
Use the tangent function to find the value of missing side.
$\tan(54^0) = \frac{h}{100}$
$1.376 = \frac{h}{100}$
$100(1.376) = 100(\frac{h}{100})$
$h = 137.6$

Example 8
A ladder that is leans up against a building rests 22 feet from the base of the building. Find the length of the ladder if the angle of elevation formed from the ladder and the ground is 70 degrees. (Round answer to the nearest tenth)



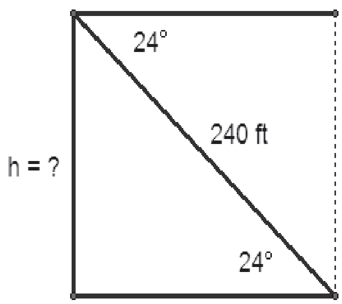
Use the sine function to find the value of missing side.
$\cos(70^0) = \frac{22}{d}$
$.342 = \frac{22}{d}$
$d(.342) = d(\frac{22}{d})$

$.342d = 22$
$\frac{.342d}{.342} = \frac{22}{.342}$
$d = 64.3ft$

1.6.6 Angle of Depression

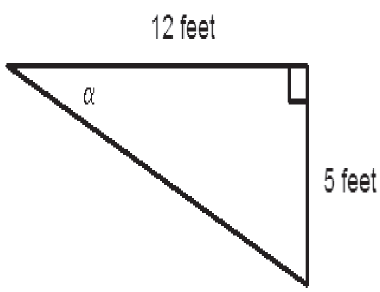
when the angle in the problem starts at the ground or line of sight and then move downward, the angle is referred to as an angle of depression

Example 9
A man stands on the edge of a cliff over looks a river. If the edge of cliff is 240 feet from the river and forms an angle of depression of 24 degrees, find the height of the cliff.



Solution
$\sin(24^0) = \frac{h}{240}$
$.407 = \frac{h}{240}$
$240(.407) = 240(\frac{h}{240})$
$h = 97.6$

Example 10
A 12 foot rope secures a row boat to a pier that is 5 ft above the water. What is the angle of formed by the rope and the water? (Angle α)

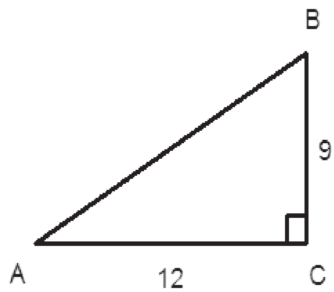


Solution
Since we know the value of the opposite and adjacent sides, we can use the tangent function to solve for the missing angle.
$\tan A = \frac{5}{12}$
$\tan A = .417$
$A = tan^{-1}(.417)$

$A = 22.6^0$

1.6.7 Finding Missing Angles and Sides

Example 10
Find the value of the missing angle A.

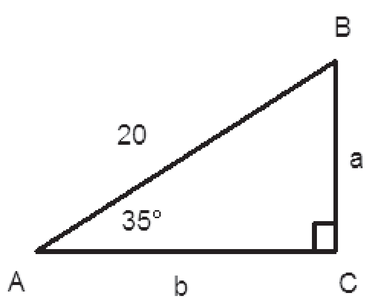


Solution
since we know the value of the opposite and adjacent sides, we can use the tangent function to solve for the missing angle.
$\tan A = \frac{9}{12}$
$\tan A = .750$
$A = tan^{-1}(.750)$
$A = 36.9^0$

1.6.8 Solving Triangles (Optional)

In the next example we will solve a right triangle. When you solve a right triangle, you find the measure of all three angles and sides.

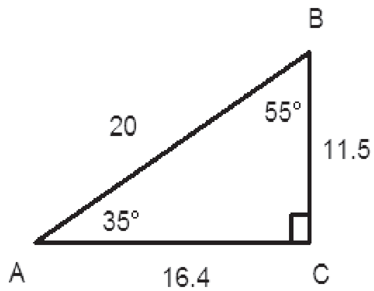
Example 11
Solve the following triangle.



Solution
As mentioned before to solve the triangle we must find the value of all the sides and angles. Since we know the value of angle A and the hypotenuse, we will use the sine function to find the value of side a.

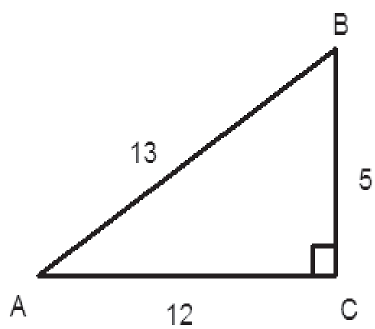
$\sin(35^0) = \frac{a}{20}$
$.574 = \frac{a}{20}$
$20(.574) = 20(\frac{a}{20})$
$a = 15.5$

Now, we can use the cosine function to find the value b.
$\cos(35^0) = \frac{a}{20}$
$.819 = \frac{a}{20}$
$20(.819) = 20(\frac{a}{20})$
$a = 16.4$
To find the value of angle B, simply add 90^0 to angle A and subtract from 180^0
$B = 180^0 - (35^0 + 35^0) = 180^0 - 125^0 = 55^0$
Here is a drawing of the triangle with all measurements of the sides and angles.

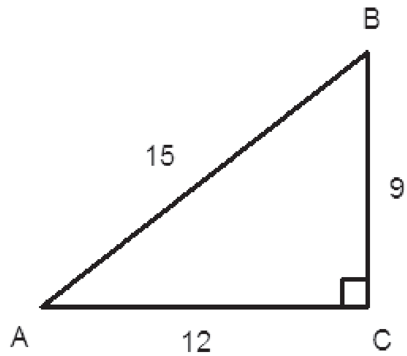


1.6.9 Exercises

1. Find the values of sin A, cos A, and tan A. (Keep answers in a fraction form)



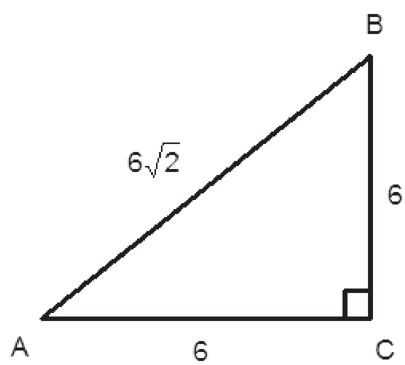
2. Find the values of sin A, cos A, and tan A. (Keep answers in a fraction form)



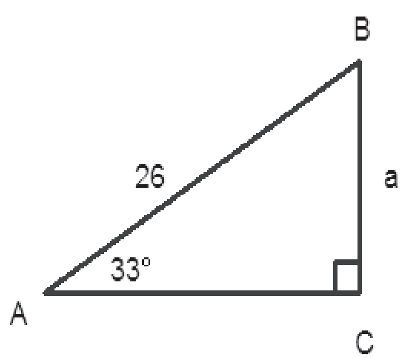
3. Use a calculator to evaluate. (Round to nearest hundredth)
   a) $\sin(25^0)$ b) $\cos(34^0)$ c) $\tan(67^0)$

4. Use a calculator to find the value of each inverse function
   a) $tan^{-1}(.897)$ b) $cos^{-1}(.453)$ c) $sin^{-1}(\frac{1}{2})$

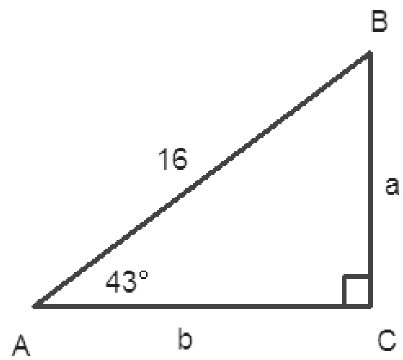
5. Use the right triangle below to find sin A, cos A, tan A, sin B, sin B, tan B



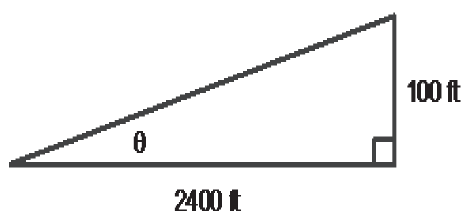
6. Find the value of a. (Round to the nearest tenth)



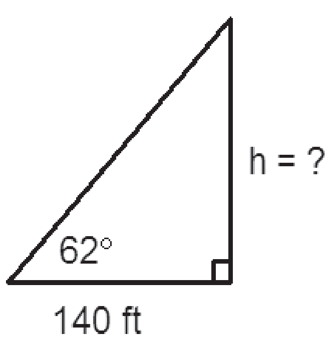
7. Find the value of the sides a and b. (Round to the nearest tenth)



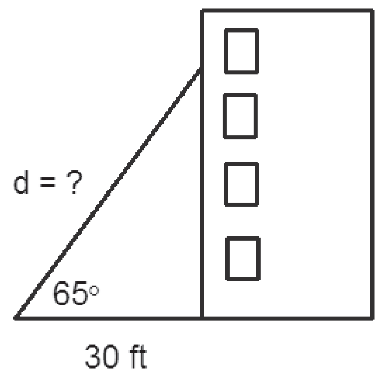
8. Find the the angle of elevation. (Angle θ)



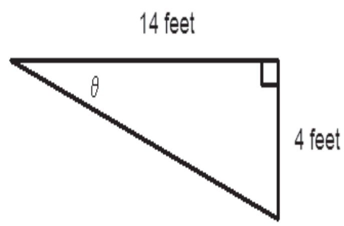
9. From a point 140 feet from the base of a cliff, the top of a cliff is seen through an angle of elevation of 62. How tall is the cliff?



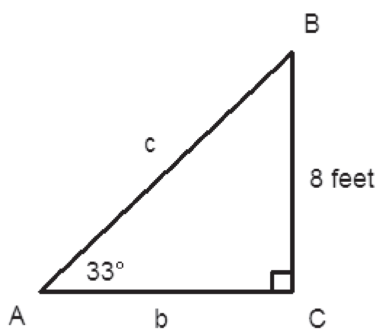
10. A ladder that is leans up against a building rests 30 feet from the base of the building. Find the length of the ladder if the angle of elevation formed from the ladder and the ground is 65 degrees.



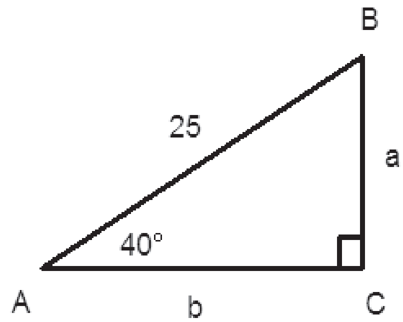
11. A 12 foot rope secures a row boat to a pier that is 5 ft above the water. What is the angle of formed by the rope and the water? (Angle θ)



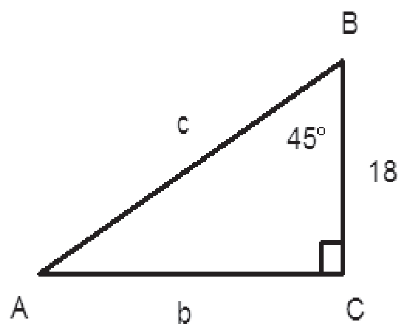
12. Find the missing sides b and c of the given triangle.



13. Solve the following triangle

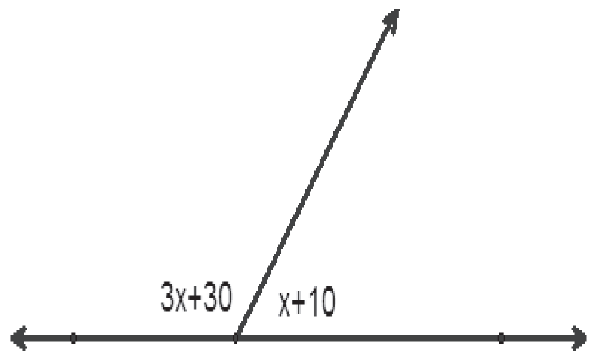


14. Solve the following triangle

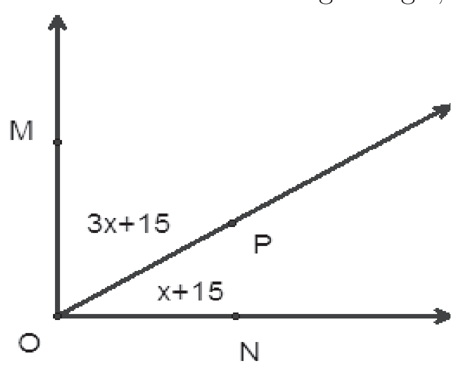


1.7 Review Exercises

1. Find the compliment of 35^0?
2. Find the supplement of 75^0?
3. Use the diagram below to find the value of x.

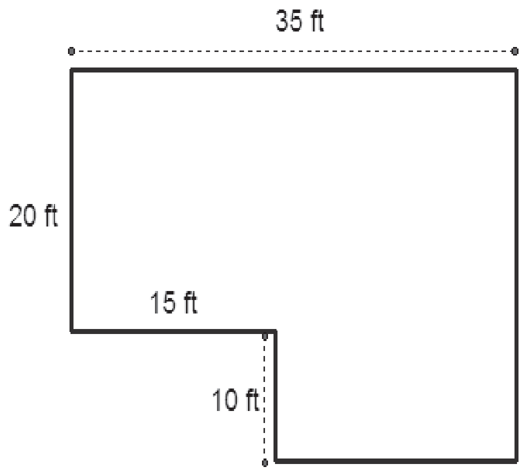


4. Given that $\angle MON$ is a right angle, find the measure of $\angle MOP$ and $\angle PON$

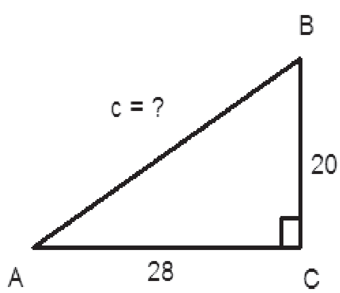


5. Find the area of a rectangular shaped lot with a length of 70 yards and a width of 35 yards.
6. You want to put down hardwood floors in your living room that is roughly 20 feet by 14 feet. If the cost of the hardwood flooring is \$4.25 per square foot, find the approximate cost not including labor to put down hardwood flooring in your living room?

7. Given the floor plans for the 1st floor of a house below, find the area or square footage on the 1st floor of this house.



8. Find the volume of a cylinder with a radius of 3 inches and height of 4 inches.
9. Find the volume of a cone with a radius of 6 centimeters and a height of 8 centimeters
10. Suppose you have a cylinder shaped hot water heater that has a height of 5 feet and a radius of 1 foot. How water can the hot water heater hold in cubic feet and gallons?
11. A fish aquarium shaped like a rectangular solid is 30 inches wide, 20 inches long, and 20 inches tall. How much volume could the fish aquarium hold?
12. Find the length of the missing side: a = 6, b = 8, and c = ?
13. Find the length of the missing side. a = 12, c = 20, b = ?
14. Find the length of the missing side.

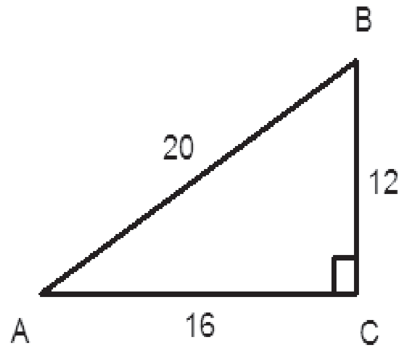


15. A rectangular shaped lot is 80 ft by 60 ft. How many feet would you save walking diagonally across the lot instead of walking length and width?
16. Use a calculator to evaluate. (Round to nearest hundredth)

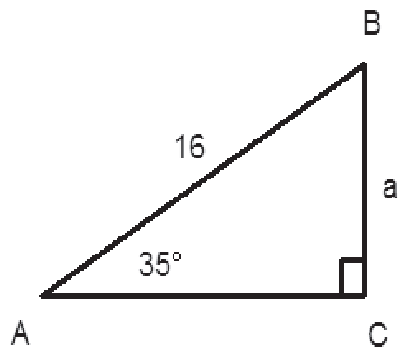
    a) $\sin(45^0)$ b) $\cos(40^0)$ c) $\tan(20^0)$
17. Use a calculator to find the value of each inverse function

    a) $tan^{-1}(.766)$ b) $sin^{-1}(.453)$ c) $cos^{-1}(\frac{1}{2})$

18. Use the right triangle below to find sin A, cos A, tan A, sin B, sin B, tan B



19. Find the value of a. (Round to the nearest tenth)



20. A ladder that is leans up against a building rests 30 feet from the base of the building. Find the length of the ladder if the angle of elevation formed from the ladder and the ground is 65 degrees.